Critical Temperature of the Ising Ferromagnet on Proximity Graphs derived from Square Lattices by Site Displacement

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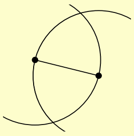
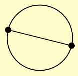
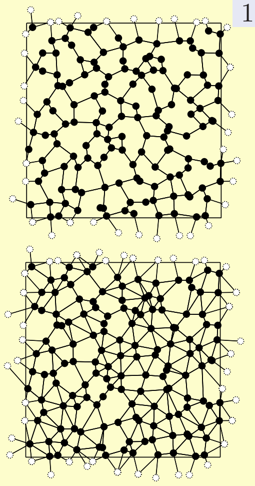
Proximity Graphs

1

- Relative Neighborhood graph (RNG) [1]
  - nodes i, j connected if there is no other node nearer to both
- Gabriel graph (GG) [2]
  - supergraph of RNG
  - nodes i, j connected if circle with diameter $\overline{ij}$ is empty
- on a square lattice both reduce to nearest neighbor relationships



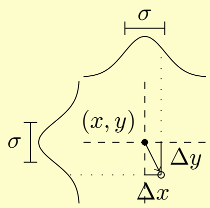




Node Displacement σ

2

- random displacement of square lattice nodes
- displacement drawn from Gaussian of width σ



Model

3

- Ising model for ferromagnets in two dimensions
- Hamiltonian: $H = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j$, $S_i = \pm 1$
- coupling J_{ij} is weaker for nodes i, j which have a greater distance d_{ij}
  $J_{ij} = e^{\alpha(1-d_{ij})}$, $\alpha = 0.5$ according to [4]
- neighborhood relationship $\langle i, j \rangle$ determined by the edges of the RNG respectively the GG
- periodic boundary conditions
- phase transition from ferromagnetic to paramagnetic at the critical Temperature T_c
- for $\sigma = 0$ this model is equivalent to a square lattice with $J_{ij} = 1$
- $\sigma \gg 1$: node positions are uniformly distributed

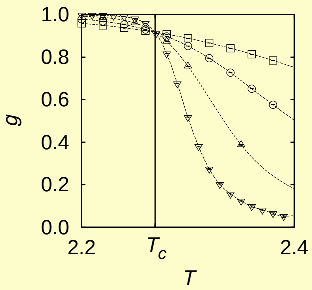
Critical Temperature T_c

4

- Binder cumulant [3]

$$g = \frac{3}{2}\left(1 - \frac{\langle m^4 \rangle}{3\,\langle m^2 \rangle^2}\right)$$

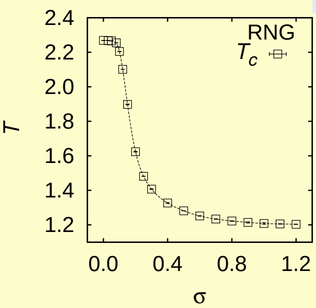
- system sizes $N = L^2$
  $L \in \{16, 32, 64, 128\}$
- g for different N cross at T_c

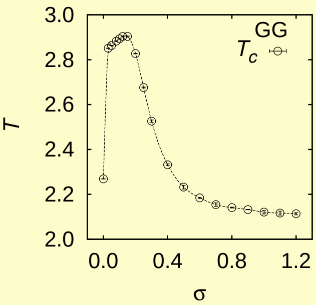


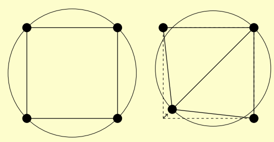
$T_c(\sigma)$

5

- T_c decreases with increasing displacement
- approaches limit of random proximity graphs for big σ
- correlated with average degree K, i.e. number of edges per node
- on GG slightest deviation from square lattice causes many new edges leading to the behavior at $\sigma \approx 0$

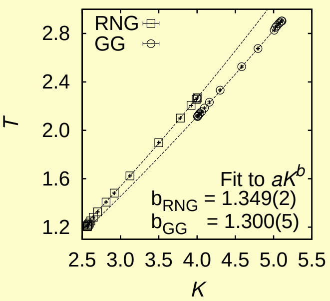






- $T_{c,\mathrm{RNG}} \leq T_{c,\mathrm{GG}}$ while $\mathrm{RNG} \subseteq \mathrm{GG}$, similar to containment theorem for percolation [5]

- approximately power law between K and T_c
- distinct exponents for both graph types
- good fit at falling T_c
- elsewhere other influences become significant, e.g. the coupling function $J_{ij}(d_{ij})$



Universality

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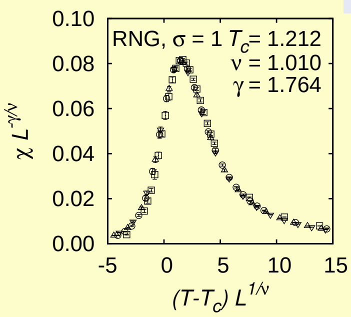
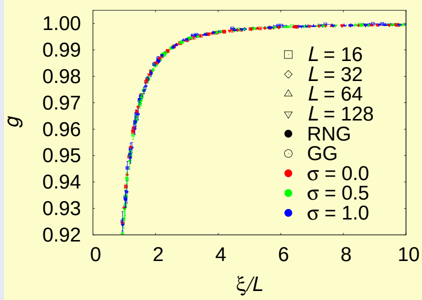
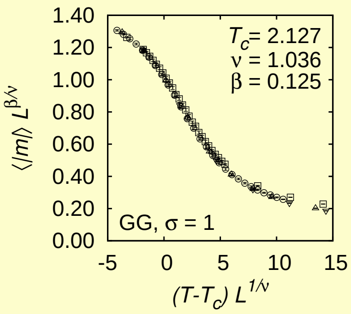
- finite size scaling analysis [6] yields same critical exponents as standard Ising ferromagnet
- g versus two-point finite-size correlation function ξ confirms same universality [7]







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